

Earth-Moon System; The Origin, Development and the Future. Mathematical-Physical Basics to the Computer Simulation of the System.

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ABSTRACT

Correct mathematical description of incoherent Earth revolving on the orbit around the Sun leads to unequivocal and logical mechanism of Moon's origin. Before 4,6 Gyr fast rotation of Earth caused outburst of the first inner core of the Earth. With fast rotation of the Earth ($T_d=4h$) the solid ferro-nickel sphere broke the thin mantle and was centrifuged to the orbit round the Earth. On the Earth's orbit the gravity force of this ferro-nickel sphere was dominant over other fragments of Earth which didn't fall back on its surface but remained on the orbit. These fragments of mantle and the outer core were pretty fast 'accumulated' on the orbit and created the natural satellite of the Earth. Presence of the Moon on the orbit slowed down the rotation of Earth above the limit value of 4 hours; that is why there was no other outburst of inner core.

Correct mathematical description of orbital motion of Earth-Moon system shows that the so called "tidal pumping of angular momentum" does not exist, as it is contradictory to the basic physical and mathematical laws.

Keywords: Earth, Moon, Origin, Celestial Mechanics, Tidal Effect.

INTRODUCTION

Few very important mathematical and logical errors in physics and celestial mechanics is the reason that until now there is no coherent and logical model of the Earth-Moon System and Solar System origin. In the considerations concerning the inner core of the Earth the fact that the Earth rotates was forgotten. It means that the rotating system of coordinates is taken as an inert one, what is an obvious mathematical error. This error causes that the centrifugal force resulting from the Earth's rotation is being omitted and an incorrect hypothesis that the interior of the Earth is spherically symmetrical is being created.

The motion of the Moon around the Earth is also considered in an inert system of coordinates and the orbital centrifugal force of the motion of the coordinates' system around the Sun is also omitted. The aim of this paper is to synthesize the nowadays known and proved facts from the history of Earth and Moon and to present a logically coherent mechanism of formation and evolution of the Earth-Moon System. The model presented below and the mathematical description may be the basis for creation a computer simulation of the origin and development of the Earth-Moon System.

THE FORCES IN THE INTERIOR OF THE EARTH

During any shift (a) (Fig.1) of the inner core from the center of the earth, two mass eccentricities Eq.(1) and Eq.(2) develop the gravity force acting between them.

$$\Delta m_{G2} = \frac{4}{3} \pi a^3 \rho_2 \quad (1)$$

$$\Delta m_{G3} = \frac{4}{3} \pi a^3 (\rho_3 - \rho_2) \quad (2)$$

ρ_2, ρ_3 - the linearised density of the outer and inner core

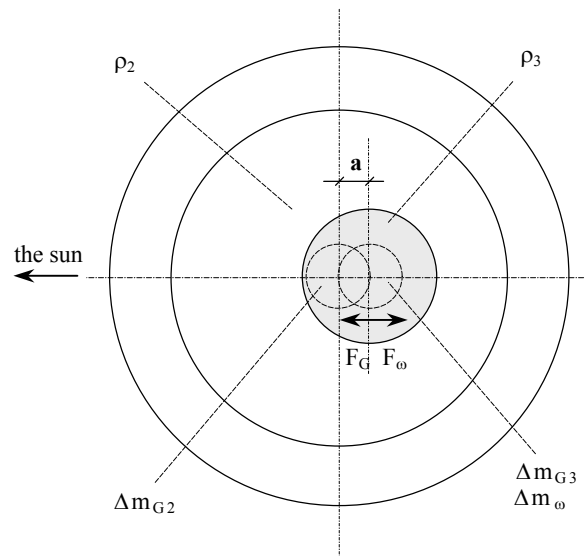


Fig. 1.: The meridional cross section of the earth
 a - shift of the inner core from the center of the Earth

In the contemporary geophysics it lingers and is repeated for years the following argumentation:

- with any shift of the inner core from the center of the Earth there is a gravitational force acting between the masses Eq.(1) and Eq.(2) (Fig.1). Due to that force the inner core would have to make vanishing oscillations of period about 5h long. These vanishing oscillations would bring the inner core back to the center of the Earth and might have been without any problems registered on the surface of the Earth. And because until now even the most sensitive gravimeters haven't registered those 5h-long oscillations, the geophysicists conclude that the Earth's inner core lies in the geometrical center of Earth and is motionless in relation to the mantle.

This conclusion is one of the greatest mistakes in the history of geophysics. The geophysicists have forgotten to take into this consideration a very important fact that **the Earth does revolve!!!** The lack of the 5h-oscillations cannot be the basis to draw conclusions concerning the position of the inner core. Rotation of the Earth determines that in the system of coordinates in the Fig.1 exists a centrifugal force F_ω that acts on the inner core in direction opposite to the gravitational force.

The position of the inner core inside the mantle is determined by the gravity force F_G and the centrifugal force F_{ω} ratio.

The centrifugal force acts in the mass system 'mantle-outer core-inner core' of the Earth on the rotational mass eccentricity:

$$\Delta m_{\omega} = \frac{4}{3} \pi a^3 \rho_3 \quad (3)$$

The centrifugal force [1] acting on rotational mass eccentricity Eq.(3) is described by the following Eq.(4):

$$F_{\omega \Delta m} = \Delta m_{\omega} \omega_R^2 a = \frac{4}{3} \pi a^3 \rho_3 \omega_R^2 a = \frac{4}{3} \pi \omega_R^2 a^4 \rho_3 \quad (4)$$

ω_R – rotational speed of the earth

The gravity force between the mass eccentricities (1) and (2) is described by the following Eq.(5):

$$F_{G \Delta m} = G \frac{\frac{4}{3} \pi a^3 (\rho_3 - \rho_2) \frac{4}{3} \pi a^3 \rho_2}{a^2} = G \frac{16}{9} \pi^2 (\rho_3 - \rho_2) \rho_2 a^4 \quad (5)$$

G - gravity constant

From the Eq.(4) and Eq.(5) we can calculate the ratio of the gravity and centrifugal forces

$$\frac{|F_{G \Delta m}|}{|F_{\omega \Delta m}|} = \frac{G \frac{16}{9} \pi^2 a^4 (\rho_3 - \rho_2) \rho_2}{\frac{4}{3} \pi \omega_R^2 a^4 \rho_3} = G \frac{4}{3} \pi \frac{1}{\omega_R^2} \left(1 - \frac{\rho_2}{\rho_3}\right) \rho_2 \quad (6)$$

The Eq.(6) allows to calculate the limit of rotational stability of the Earth, i.e. the parameters of the mass system 'mantle-outer core-inner core', with which the centrifugal force acting on the inner core would be equal to the gravitational force.

$$\frac{|F_{G \Delta m}|}{|F_{\omega \Delta m}|} = 1 \quad (7)$$

After substitution $\omega_R = 2\pi/T$ and Eq.(7) in Eq.(6) and after simple transformations we obtain

$$T = \frac{1}{\sqrt{\frac{1}{3\pi} G \left(1 - \frac{\rho_2}{\rho_3}\right) \rho_2}} \quad (8)$$

From the Eq.(8) we can see that the limitary period of the Earth's rotation depends only on the densities ρ_1 and ρ_2 . After substitution to the Eq.(8) the gravity constant

$$G = 6,6726 \cdot 10^{-8} \text{ cm}^3/\text{g} \cdot \text{s}^2$$

and the contemporary magnitudes of the inner and outer core densities (PREM model)

$$\rho_2 = 12,1 \text{ g/cm}^3, \rho_3 = 12,8 \text{ g/cm}^3$$

we obtain

$$T = 14610,036 \text{ [s]} = 4,058 \text{ [h]} \quad (9)$$

The value from Eq.(9) means that with a day length of $T = 4,058$ hours our today's Earth would rotate at the limit of rotational stability.

FORMATION OF THE MOON

From the astronomical measurements and paleontology research it appears that rotation of the Earth undergoes slowing down. Assistant Professor S. Maj from the Institute of Geophysics of the Polish Academy of Sciences wrote [2]:

"(...) during 4,6 billion years of the Earth's history Earth's day lengthened for about 20 hours."

It means that the Earth's day at the beginning of the history of the Earth lasted about 4 hours then, after different energy dissipation, it has lengthened to the present 24 hours.

Research on the rocks brought from the Moon show that the oldest regions on the Moon were in the liquid state 4.6 billion years ago. This point can be approximately taken as the moment of Moon's formation. With a good approximation we can also assume that the differences in densities of the inner and outer core of the young Earth (4,6 billion years ago) were similar to the present ones.

So we have the following convergence of proved facts:

- 4,6 billion years ago the Earth's rotation was so fast that the centrifugal force acting on the inner core equaled the gravitation force. So the mantle-core mass system rotated on the limit of rotational stability.

- in the same time (4,6 billion years ago) the Moon was also created.

This convergence of facts is not accidental. This time relationship proves that, approx. 4,6 billion years ago, by the quick rotation of the young Earth its first inner core was centrifuged from the thin mantle into the space. With fast rotation of the Earth the solid ferro-nickel sphere broke the thin mantle and was centrifuged to the orbit round the Earth. On the Earth's orbit the gravity force of this ferro-nickel sphere was dominant over other fragments of Earth which didn't fall back on its surface but remained on the orbit. These fragments of mantle and the inner core were pretty fast 'accumulated' on the orbit and created the natural satellite of the Earth. The surface of the contemporary Moon is formed of fragments coming from different depths of the young Earth, which explains the so called 'bulk composition' of the Moon's surface.

After the outburst of the first inner core the earth closed its mantle again. The place where the mantle was broken remained after the closing lower than the rest of the Earth's surface. Water flowed to this region of the Earth and created the Primal Pacific Ocean. The outburst scenario explains the until now unexplained fact why the crust of the Earth under the continents is much thicker than under the oceans. The continents are the remainders of the Earth's crust that had been created from the very beginning of the Earth's history, whereas the oceanic crust was created again after the outburst of the first inner core.

During the next few million years there was a new inner core created inside the Earth.

Presence of the Moon on the orbit slowed down the rotation of Earth above the limit value of 4 hours; that is why there was no other outburst of inner core. The same forces which 4.6 billion years ago were the reason of creation of the Moon, nowadays are the reason of "only" the deformations of our planet and the continental drift on its surface [3].

THE SITUATION AFTER THE FORMATION OF THE MOON

The formation of the Moon on the Earth's orbit had changed determinative the parameters of the Earth's motion around the Sun. During the time when Earth revolved around the Sun as a single celestial body the ecliptic was the plane of Earth's center of gravity orbit. After the formation of the Moon the orbit around the Sun is being defined not by the Earth only but by the center of gravity of the Earth-Moon System. According to the Newton's Modification of Kepler's Third Law Earth and Moon revolve round their common center of gravity, which means that the distance of the Earth and Moon from the Sun is not constant. Contemporarily the center of gravity of the Earth-Moon System is shifted about 4500 km from the center of Earth. Therefore the distance of the Earth from the Sun changes of about $\pm 4\,500$ km and the distance of the Moon from the Sun of about $\pm 3\,000\,000$ km. Both the gravity force and the orbital centrifugal force depend on the distance from the Sun.

That is why the assertion that the Earth and Moon are "free falling" on the orbit around the Sun is entirely false!

The only "free falling" point on the orbit around the Sun is the gravity center of the Earth-Moon System.

Earth and Moon Fig2. make a double, coupled Kepler's movements:

- Earth and Moon revolve around their common center of gravity
- the center of gravity of the Earth-Moon System revolves around the Sun

The description of Earth and Moon motions around the Sun in the system of coordinates with the center in the center of gravity of the Solar System leads to unstable equations due to the radius of the Moon's orbit R_M and Earth's orbit R_E around their common center of gravity. From those equations it follows that the distance Earth-Moon (generally: planet-satellite distance) always increases in case of such doubly coupled Kepler's motion. The velocity of alteration of the distance increases together with increase of the radius of Moon's revolution around Earth and the radius of revolution around the Sun ratio (R_M/R_E). It means that the further the Moon is from Earth, the faster it moves away, until in the future it will move to its own orbit around the Sun through the Lagrange's point $L1$.

To simplicate the drawing, in the Fig.2 there is only shown Moon's motion around the centre of gravity of the Earth-Moon system as the motion around the Earth. All the below considerations and equations consider in the same form the Earth's synchronous motion around the centre of gravity of the Earth-Moon system. Earth's motion is shifted of 180° in relation to Moon's motion.

In Fig.2 we can see that both the Moon's orbital velocity around the Sun and its distance from the Sun depend on Moon's position (angle α) on the orbit around the Earth. On the outer side of the orbit ("full moon") Moon moves on a longer radius and with increased velocity than the centre of gravity of the Earth-Moon system. It means that Moon overtakes the centre of gravity of the Earth-Moon system. The centrifugal force of its motion around the Sun is greater than Sun's gravity force.

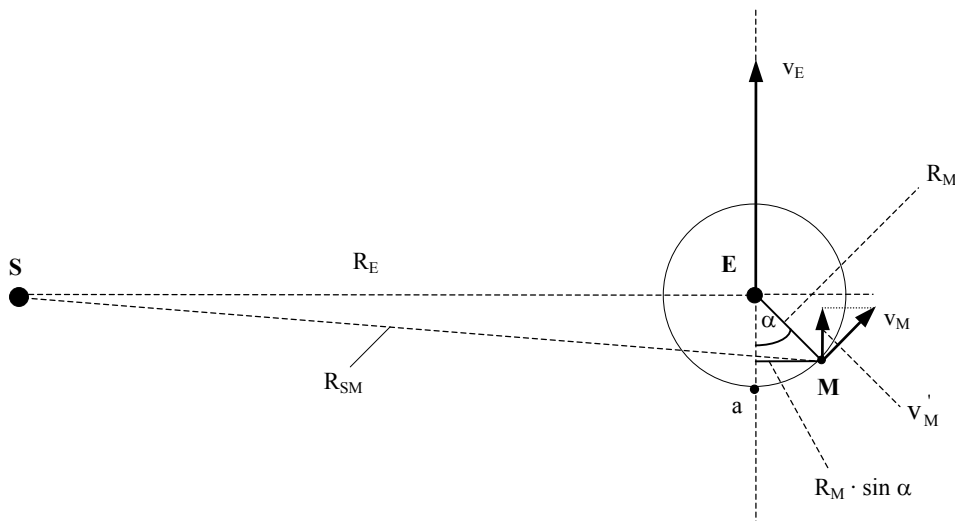


Fig.2 The Sun-Earth-Moon (SEM) system of masses and relative motions of the System's components (radii and lengths of vectors do not correspond with the actual proportions of the system)

S - Sun; **E** - Earth; **M** - Moon

R_E - mean radius of the Earth's orbit around the Sun

R_M - mean radius of the Moon's orbit around the Earth

R_{SM} - the Sun-Moon distance

v_E - mean orbital velocity of Earth around the Sun

v_M - orbital velocity of Moon around the Earth

α - angle describing Moon's position on the orbit around the Earth

v'_M - component of vector of Moon's motion around the Earth parallel to the vector of Earth's motion around the Sun

On the inner side of the orbit ("new moon") Moon moves slower than the centre of gravity of Earth-Moon system. It means that the Sun's gravity force is greater than the centrifugal force of the orbital motion.

The parallel component v'_M of Moon's velocity is:

$$v'_M(t) = v_M \cdot \sin \omega_M t \quad (10)$$

where:

$$\alpha = \omega_M t$$

ω_M – angular velocity of Moon's revolution around the Earth.

In Fig.2 we can see that the Moon's distance from the Sun and its orbital velocity around the Sun are, appropriately:

$$R_{SM}(t) = R_E + R_M \cdot \sin \omega_M t \quad (11)$$

$$v_{SM}(t) = v_E + v_M \cdot \sin \omega_M t \quad (12)$$

From Eq.(11) and Eq.(12) we can elaborate centrifugal force and the gravity force of Moon's motion around the Sun:

$$F_{\omega SM}(t) = \frac{M_M \cdot (v_E + v_M \cdot \sin \omega_M t)^2}{R_E + R_M \cdot \sin \omega_M t} = \frac{M_M \cdot (v_E + v_M \sin \omega_M t)^2}{R_E \left(1 + \frac{R_M}{R_E} \cdot \sin \omega_M t\right)} \quad (13)$$

$$F_{GSM}(t) = G \frac{M_{\odot} \cdot M_M}{(R_E + R_M \cdot \sin \omega_M t)^2} = G \frac{M_{\odot} \cdot M_M}{\left[R_E \left(1 + \frac{R_M}{R_E} \sin \omega_M t\right)\right]^2} \quad (14)$$

From Eq.(13) and Eq.(14) we can see that with a correct description of Earth-Moon system motion around the Sun both the gravity force and the orbital centrifugal force oscillate

(term $\sin \omega_M t$) around the constant value of these parameters in the centre of gravity of the system. The result is that the excess of Sun's gravity on the inner side of orbit and the excess of the orbital centrifugal force on the outer side of orbit increase the distance of Earth and Moon from their common centre of gravity.

Precise calculations and integrating the paired motion of the Moon and Earth around their centre of gravity and around the Sun would go beyond the range of this article. The aim of the above elaboration is to show logical mechanism of increase of Earth-Moon distance.

The presented in contemporary physics mechanism of "tidal pumping" of angular momentum from Earth to the Moon is logically and mathematically incorrect. The logical discrepancy here is that the deformations of Earth's body, that are caused by Moon itself, were to increase the distance between these celestial bodies. That would mean that the result of gravity force, i.e. mutual attraction of the celestial bodies, was increasing the distance, i.e. mutual repulsion. This theorem is equivalent to the one that it is possible to lift oneself from the ground by pulling oneself's hair.

Mathematical discrepancy of this mechanism is that in Newton's Modification of Kepler's Third Law in the description of Moon's orbital motion around Earth there is arithmetical sum of

Earth's and Moon's masses ($M_E + M_M$) what means that Earth and Moon revolve around their common centre of gravity.

The fact that both celestial bodies revolve around their centre of gravity means that there are three significant points in the space:

- Earth's centre of gravity
- Moon's centre of gravity
- the common centre of gravity of the Earth-Moon system

Both orbits around the common centre of gravity describe the centres of gravity (!) of Earth and Moon. It means that through the common centre of gravity of the Earth-Moon system flows no information concerning the inner structure of the "orbital partner" on the other side of the centre of gravity. Both celestial bodies "don't see" each other.

The direct consequence of this fact is that **the "tidal pumping of angular momentum" from Earth to Moon does not exist!** The whole angular momentum of Earth was "used" in the history of Earth for the heat of internal tidal friction, plate tectonics and to generate the magnetic field.

The density paradox of the moon:

The mean density of the moon is

$$\rho_M = 3,344 \text{ g/cm}^3$$

The density of rocks which the Apollo missions brought to the earth from the moon ranged between 2.8 – 3.4 g/cm³. With an increasing depth also the density inside a celestial body is growing. From the physical and mathematical standpoint, it is, therefore, not possible that the mean density of the moon is lower than the smallest value of the density of the outer layers of the moon crust.

If the moon had developed from the first core and fragments of the earth its mean density would have to be more or less identical with the density of today's earth. However, the mean density of the moon is only 60.5% of the mean density of the earth.

There is only one explanation for this paradox. The moon must have hollow spaces inside which also confirms the herein presented process of the development of the moon. During its development the moon was rotating only very slowly. The first inner core of the earth pressed the still liquid lava through the young and thin crust of the moon towards outside. In this way the maria on the moon and the empty spaces inside the moon developed. Seismograms of moon-quakes lasting up to 100 min, clearly demonstrate the existence of hollow spaces inside the moon. That explain also the offset of the center-of-mass and center-of-figure of the Moon.

The inclination of the Moon's orbit

The inclination of the Moon's orbit to the plane of ecliptic causes that in the sum of the orbital forces there appear components F_P (Fig.3) perpendicular to the ecliptic. The components F_P for Earth and Moon are directed in opposite directions and cancel each other in the gravity center of the system. The basic condition of Kepler's orbit:

$$F \times R = 0$$

is therefore fulfilled only for the gravity center of the Earth-Moon system. The Earth-Moon system reacts to the perpendicular components F_P as the well known in physics peg-top, it evades their action, what leads to a 18,6 year rotation of Moon's orbit.

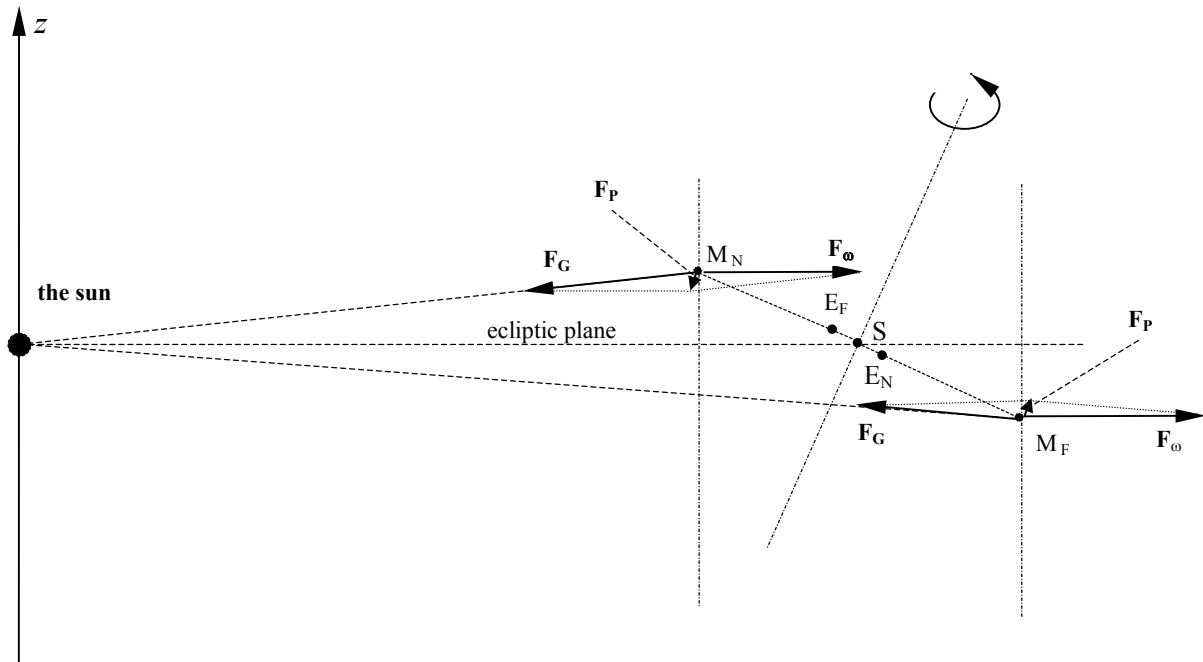


Fig. 3 Vectors of gravity force and the orbital centrifugal force acting on the Moon.
 M_N, E_N - positions of the Moon and Earth for the 'new moon'
 M_F, E_F - positions of the Moon and Earth for the 'full moon'
 S - center of mass of the Earth-Moon System
 $F_G; F_\omega$ - gravity force and centrifugal force
 F_P - component of the orbital forces perpendicular to the ecliptic

CONCLUSIONS

As we can see, the correct mathematical description of the rotating Earth leads to a coherent and logical mechanism of formation of the Moon. The outburst scenario makes a logical sequence of correlated facts of many observations that in geophysics and astronomy have not found explanation or are called 'mysteries'.

The plate tectonics drive mechanism and the drift of continents are the direct continuation of the Moon's formation mechanism. The differences in the thickness of the earth's crust are also the direct consequence of the outburst of the first inner core and breaking out a part of the primal Earth's crust.

The outburst scenario is also a simple and logical explanation for the mysterious 'late heavy bombardment'; quotation [4]:

"The late heavy bombardment occurs relatively late in the accretionary history of the terrestrial planets (...) An essential requirement of any explanation for the late heavy bombardment is that the impactors be "stored" somewhere in the solar system until they are suddenly unleashed about 4.0 Gyr ago (...). A plausible explanation for the late heavy bombardment remains something of a mystery."

From the above presented mechanism it occurs that the impactors were not "stored" anywhere but were created just in the final stage of planet's accretion. The impactors were formed

as debris when the first inner core of the Earth was big enough, broke the thin crust and moved away to the orbit.

As it is concluded from the above considerations large fragments of celestial mechanics must be completed. We must start from where Sir Isaac Newton and Johannes Kepler finished their works centuries ago. Precise mathematical models of the above presented mechanisms must take into consideration many of the static and dynamic parameters of the Earth-Moon System that until now were omitted.

The calculations will surely be very complicated but obtaining a logical picture of a fragment of history of the Solar System is undoubtedly worth it.

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